RANDOM WALK APPROACH TO THE PROBLEM OF IMPULSE NOISE REDUCTION

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Abstract

In the presented paper a new approach to the problem of noise reduction of gray and colour images has been presented. The new method is based on the concept of a virtual particle performing a random walk on the image lattice, with transition probabilities defined by the median distribution. The major advantage of this algorithm, is that it filters the image, while preserving the image structures. This new algorithm is extremely fast and can be applied to the gray scale and colour images working on the RGB image components.

1 NOISE REDUCTION FILTERS

1.1 Noise reduction in gray scale images

The noise suppression of digital images is one of the major problems of the low level image processing. An overview of the existing methods can be found in [1, 2, 3, 4, 5, 6, 7].

The additive Gaussian noise can be attenuated relatively well, using a whole variety of different algorithms [8, 9, 10, 11, 12, 13, 14]. These algorithms however, are not suitable for filtering the so called *salt and pepper* noise. This kind of noise is frequently eliminated using algorithms based on the rank transformations, defined using an ordering operator, the goal of which is the transformation of the set of pixels $B(x, y) \in W(i, j)$ lying in a given filtering window W of the size $n_1 \times n_2$ with the center at (i, j) into a monotonically increasing sequence

$$\{B(i-k, j-l)\},\ k=-n_1,\dots,n_1,\ l=-n_2,\dots,n_2\} \rightarrow \{B_1,\dots,B_N\},B_k \leq B_{k+1},\ k=1,\dots,N-1 \quad (1)$$

where $N = (2n_1 + 1)(2n_2 + 1)$ is the number of pixels belonging to $W(i, j)$.

In this way the rank operator is defined on the ordered values from the set $\{B_1, B_2, \dots, B_N\}$ and has the form

$$T(i,j) = \frac{1}{Z}\sum_{k=1}^{N} \varrho_k\, I_k \quad (2)$$

where Z is the normalizing constant and ϱ_k are the weighting coefficients. Taking appropriate ranking coefficients allows the defining of a variety of useful operators [4, 7, 6].

The sequence

- $\{1, 1, \dots, 1\}$ corresponds to the moving average operator,
- $\{0, \dots, 0, \varrho_M = 1, 0, \dots, 0\}$, where $M = \frac{N+1}{2}$ generates the median,
- $\{0,\dots,0,\varrho_{M-\alpha} = 1 = \dots = \varrho_M = \dots = \varrho_{M+\alpha} = 1, 0, \dots, 0\}$, $0 \leq \alpha \leq M$ defines the α-trimmed mean, which is a compromise between the median ($\alpha = 0$) and the moving average ($\alpha = M$),
- $\{\varrho_1 = 1, 0, \dots, 0, \varrho_N = 1\}$ determines the so called mid-range filter.

The median filter is one of the most commonly used nonlinear filters. It has the ability of attenuating strong impulse noise, while preserving image edges. Its major drawback however, is that it wipes out structures, which are of the size of the filter window and this effect causes that the texture of a filtered image is strongly distorted. In this way the side effect of noise elimination is very often the blurred image texture. Additionally the sorting process required by the median is time expensive and that is why recursive procedures and different kinds of so called pseudo-medians were proposed [4, 15].

For instance the pseudo-median MED_S is defined as

$$\begin{aligned} m_l &= \mathrm{MED}\{B(i-k,j-l),B(i-k+1,j-l),\ldots,B(i+k,j-l)\} &(3)\\ \mathrm{MED}_S &= \mathrm{MED}\{m_{j-z},m_{j-z+1},\ldots,m_{j+z}\}, \quad -z\le k,l\le z=\frac{N-1}{2} &(4)\end{aligned}$$

Another example is the max/min -median filter , which preserves such structures as lines and corners [16, 17]. Taking the filter window $W(i,j)$ the following parameters $\mu_i,\ i=0,\ldots,4$ are computed

$$\begin{aligned} \mu_1 &= \mathrm{MED}\{\ B(k,1),B(k,2),\ldots,B(k,l),\ldots,B(k,n)\ \} &(5)\\ \mu_2 &= \mathrm{MED}\{\ B(1,l),B(2,l),\ldots,B(k,l),\ldots,B(l,n)\ \} &(6)\\ \mu_3 &= \mathrm{MED}\{\ B(1,1),B(2,2),\ldots,B(k,l),\ldots,B(n,n)\ \} &(7)\\ \mu_4 &= \mathrm{MED}\{\ B(1,n),B(2,n-1),\ldots,B(k,l),\ldots,B(n,1)\ \} &(8)\\ \mu_0 &= \mathrm{MED}\{\ B(i,j):\ (i,j)\in W(k,l)\ \} &(9)\end{aligned}$$

and additionally

$$\mu_{\max}=\max\{\ \mu_1,\mu_2,\mu_3,\mu_4\ \}, \qquad \mu_{\min}=\min\{\ \mu_1,\mu_2,\mu_3,\mu_4\ \} \tag{10}$$

Then the max/min operator is defined as

$$T(i,j)=\begin{cases}\mu_{\max} &: \quad |\,\mu_{\max}-\mu_0\,|\ge|\,\mu_0-\mu_{\min}\,|\\ \mu_{\min} &: \quad |\,\mu_{\max}-\mu_0\,|<|\,\mu_0-\mu_{\min}\,|\end{cases} \tag{11}$$

One of the derivatives of the median is the *weighted median filter, WMF* [18, 19]. If the output $T_{MED}(x)$ of the pixels lying in the filter window W, with domain D is denoted as

$$T_{MED}(x)=MED\{B(x+y),\ y\in D\} \tag{12}$$

then the *WMF* can be defined as

$$T_{WMF}(x)=MED\{w(y)\diamondsuit B(x+y),\ y\in D\} \tag{13}$$

where $\diamondsuit$ denotes the repetition or duplication operator and $w(y)\in\mathbf{N}$ are the weighting coefficients. In case of the *WMF* filter different emphasis is assigned to different samples in the filter window by repeating them suitable many times. The main advantage of this kind of filtering is its capability of suppressing noise and avoiding the blurring effect. Its main drawback, however is that it is very difficult to choose appropriate weighting coefficients [20].

One of the many median derivatives is the so called *KNNM, k-nearest neighbour median filter* [21]. It is defined using the Φ function defined as $R=\Phi[r,N,\{B_1,B_2,\ldots,B_N\}]$, where R is the r-th smallest element from the ordered sequence $\{B_1,B_2,\ldots,B_N\}$ from a given window $W(i,j)$. Then the output of *KNNM* is defined as follows

$$T(i,j)=\Phi[g,k,\{B_s,B_{s+1},\ldots,B_{s+G-1}\}] \tag{14}$$

where $g=\mathrm{INT}\{\frac{k+1}{2}\}$, $1\le k\le N$ is a certain odd number, $s=\min\{\ N-G+1,\max\{1,\rho-g+1\}\}$, ρ is the rank of the value $B(i,j)$ of the pixel (i,j) and N is the overall number of pixels in the filter window $W(i,j)$.

The *KNNM* first selects k samples, which have the rank values that are nearest to the centre pixel of the filter window and then it takes the median of the k samples as an output. The *KNNM* removes impulsive noise with less distortion of image details, than the standard median does.

Another frequently used operator *MADTM (median of absolute differences trimmed mean-filter)* [7, 22], is described as follows. At first the median ν_1 of the pixels from the window $W(i,j)$ is obtained

$$\nu_1=\mathrm{MED}\{\ B(k,l):(k,l)\in W(i,j)\ \}=\Phi[w,N\{B_1,\ldots,B_N\}],\ w=\frac{N+1}{2} \tag{15}$$

and then the median ν_2 of the absolute differences of the pixels from $W(i,j)$ from ν_1 is computed

$$\nu_2 = \Phi[w, N\{|B_1 - \nu_1|, \ldots, |B_N - \nu_1|\}], \quad w = \frac{N+1}{2} \tag{16}$$

The output of the *MADTM* operator is defined as

$$T(i,j) = \frac{1}{Z} \sum_{(k,l)\in W(i,j)} B(k,l) : \{|B(k,l) - \nu_1| \leq \nu_2\} \quad Z = \mathrm{card}\{(k,l) : |B(k,l) - \nu_1| \leq \nu_2\} \tag{17}$$

In this way $T(i,j)$ is the mean value of the absolute differences, whose values do not exceed the median more than ν_2. The *MADTM* transformation resembles to some extent the α-trimmed mean filter.

1.2 Noise removal from colour images

Most of noise removal methods were originally devised for monochromatic images. In the majority of cases it is possible to operate on colour channels separately and in this way the gray scale image algorithms can be applied directly for the colour images working independently on separate channels. Unfortunately, these methods are mostly in danger of introducing shifts in colour and loss of chromatic information. That is why the techniques which work directly with *vector-valued* pixels are more effective with this regard.

The rank order filters based on a concept of the input ordering cannot be simply extended to multi channel data. Therefore, a number of algorithms using various methods of multivariate data ordering have been proposed [23, 24, 25, 26]. The best known vector order statistic filter is the so called *Vector Median Filter, VMF*. The vector median is defined as [23, 24, 26, 27] $VMF(\mathbf{x}) \in (\mathbf{x}_i,\ i = 1, \ldots, N)$, with

$$VMF(\mathbf{x}) = \left[\sum_{i=1}^{N} ||VMF(\mathbf{x}) - \mathbf{x}_i|| \leq \sum_{i=1}^{N} ||\mathbf{y} - \mathbf{x}_i||, \quad \forall \mathbf{y} \right] \tag{18}$$

where || denotes a norm in a given colour space and $\mathbf{x}_i$ and $\mathbf{y}$ are the vectors (pixel values) from a given filter window.

Assuming that a window W of finite size N is available, let the noisy image vectors inside this window be denoted as $\mathbf{x}_j,\ j = 1, 2, \ldots, N$. If $\rho(\mathbf{x}_i, \mathbf{x}_j)$ is a measure of distance between vectors $\mathbf{x}_i$ and $\mathbf{x}_j$ then the scalar quantity $d_{(i)} = \sum_{j=1}^{N} \rho(\mathbf{x}_i, \mathbf{x}_j)$ is the distance associated with the noisy vector x_i inside the processing window of length N. Assuming that an ordering of d_i's

$$d_{(1)} \leq d_{(2)} \leq \ldots \leq d_{(N)} \tag{19}$$

implies the same ordering of $\mathbf{x}_i$'s

$$\mathbf{x}_{(1)} \leq \mathbf{x}_{(2)} \leq \ldots \leq \mathbf{x}_{(N)} \tag{20}$$

then VMF defines the vector $\mathbf{x}_{(1)}$ as the filter output. In most cases the VMF is defined using the measure

$$d_p(i,j) = \sum_{k=1}^{m} \left| x_i^k - x_j^k \right| \tag{21}$$

where m is the dimension of vector $\mathbf{x}_i$ (for RGB images $m = 3$) and x_i^k is kth element of $\mathbf{x}_i$ which is utilized.

2 NEW ALGORITHM OF NOISE ELIMINATION

The output of the average $\hat{B}(i,j)$ and median $\tilde{B}(i,j)$ of pixels surrounding the point (i,j) can be treated as values which minimize the appropriate cost functions

$$\hat{B}(i,j) = \arg\min_{\Theta} \left\{ \sum_{u=-n}^{n} \sum_{v=-n}^{n} [B(i+u, j+v) - \Theta]^2 \right\} \tag{22}$$

$$\tilde{B}(i,j) = \arg\min_{\Theta} \left\{ \sum_{u=-n}^{n} \sum_{v=-n}^{n} |B(i+u, j+v) - \Theta| \right\} \tag{23}$$

The new method of noise reduction is based on a concept of randomly walking particle, which moves on the image lattice. The probabilities $P_{ij,kl}$ of transition from the point (i,j) to (k,l) has been derived from the median distribution which describes the behaviour of a particle in a potential well

$$P_{ij,kl} = \frac{\exp\{-\beta|B(i,j) - B(k,l)|\}}{Z_{i,j}} \tag{24}$$

where

$$Z_{i,j} = \sum_{(k,l)\sim(i,j)} \exp\{-\beta|B(i,j) - B(k,l)|\} \tag{25}$$

and β is the temperature parameter, $\sim$ denotes the neighbourhood relation and Z is the normalizing constant - statistical sum of the system.

The new filter assigns to each image point (i,j) the value $B^*(i,j)$, which minimizes the probability, that the virtual jumping particle will not remain at its temporary position, but will jump to one of its neighbours. The minimization of the probability $P_{ij,kl}$ is equivalent to the maximization of the statistical sum $Z(i,j)$ and therefore

$$B^*(i,j) = \arg\max_{\Theta} \left\{ \sum_{u=-1}^{1} \sum_{v=-1}^{1} \exp\{-\beta|B(i+u, j+v) - \Theta|\} \ , \quad |u| + |v| \neq 0 \right\} \tag{26}$$

Since the statistical sum Z depends on the absolute value of the difference of the gray scale values of the neighbouring points, therefore only the gray tones from the neighbourhood are able to minimize Z (cost function). This increases the computational efficiency of the presented algorithm.

The colour version of our algorithm is very similar. At the beginning we applied the gray scale filter separately for each of the RGB channels, and then the more efficient vector version of our filter was implemented.

The statistical sum $Z\,(i,j)$ and the filter output $\mathbf{B}^*\,(i,j)$ are as follows

$$Z(i,j) = \sum_{(k,l)\sim(i,j)} \exp\left\{-\beta \left\|\mathbf{B}(i,j) - \mathbf{B}(k,l)\right\|\right\} \tag{27}$$

$$\mathbf{B}^*\,(i,j) = \arg\max_{\mathbf{\Theta}} \left\{ \sum_{u=-1}^{1} \sum_{v=-1}^{1} \exp\left(-\beta \left\|\mathbf{B}(i+u, j+v) - \mathbf{\Theta}\right\|\right) , \ |u| + |v| \neq 0 \right\} \tag{28}$$

where "$\|\cdot\|$" denotes

$$\|\mathbf{x}_{i,j} - \mathbf{x}_{k,l}\| = \left(|r\,(i,j) - r\,(k,l)| + |g\,(i,j) - g\,(k,l)| + |b\,(i,j) - b\,(k,l)|\right) \tag{29}$$

or

$$\|\mathbf{x}_{i,j} - \mathbf{x}_{k,l}\| = \sqrt{[r(i,j) - r(k,l)]^2 + [g(i,j) - g(k,l)]^2 + [b(i,j) - b(k,l)]^2} \tag{30}$$

The gray scale algorithm presented in this paper can be seen as a modification and improvement of the commonly used median filter. Although, there exists a lot of different kinds of filters, which seek to overcome the drawbacks of the median, the novel approach seems to be interesting as the filter is not invasive, in the sense that it does not destroy unnecessarily the structure of the image as the median and its derivatives do.

This effect is shown in Fig. 1, where the non invasive nature of the presented algorithm can be clearly seen, when compared with the median filter. The noise to signal ratios computed as

$$SNR = 10\log_{10} \left\{ \frac{255^2}{\sum_{i,j} [T(i,j) - B(i,j)]^2} \right\} \tag{31}$$

(**T** is the reconstructed image and **B** is the original test image) were $SNR = 27.93$ dB for the new method and $SNR = 25.77$ for the median. The slight increase of the SNR is however quite visible to human observer.

Satisfying results has been also achieved, when using the described procedure for the filtering of noisy colour images. Results achieved for colour images prove the superiority of the vector filtering approach. As expected, after filtration performed separately on each of the RGB channels, colour shifts in some regions of the image were observed.

The performance of the colour version of the new filter is depicted in figures 2 and 3. Examples of the performance of the colour version of the new noise reduction filter can be evaluated visiting our web page at address: http://plum.ia.polsl.gliwice.pl/∼marek.

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Figure 1: Comparison of the efficiency of the median and the new algorithm when applied on the noisy test image. **top left**) *BRIDGE*, **top right**) image distorted by 10 % impulse ”salt & pepper” noise, below the filtration using the new method in comparison with the 3× median (2 iterations), below all pixels whose gray scale values were changed when applying both filters on test image are marked black, the pixels with unchanged values after filtration are white.

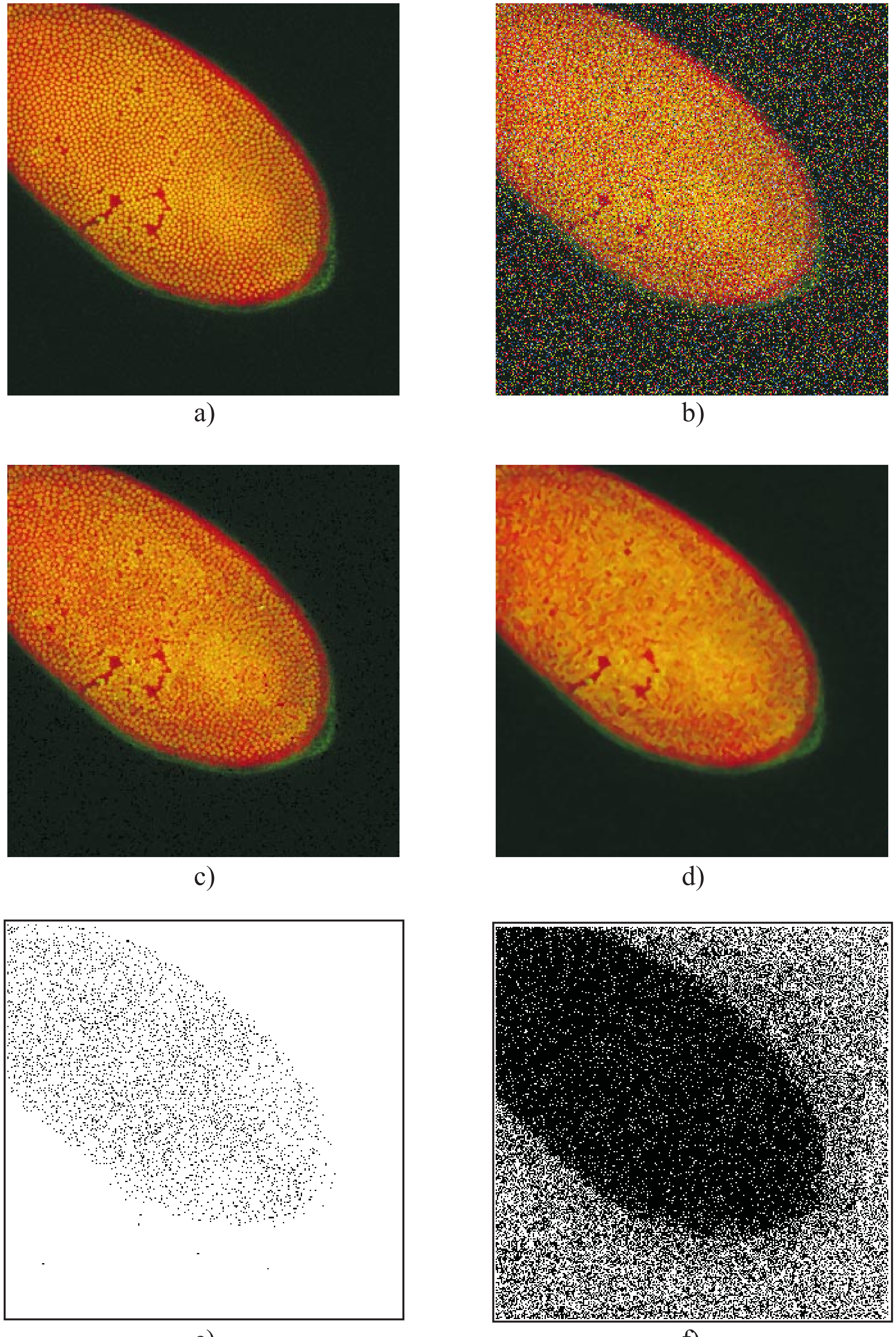

Figure 2: Comparison of the effect of the median and the new filter when applied on the noisy test image. **a**) test image, **b**) image distorted by 20 % impulse ”salt & pepper” noise added to each RGB channels, **c**) noisy image filtered with the new method (ordinary RGB algorithm, $\beta = 2$, 4 iterations), **d**) noisy image filtered with 3×3 median - 3 iterations, **e**) all pixels whose colour values were changed by our filter when applied on the test image **a**) are marked black, the pixels with unchanged values after filtration are white, **f**) the same when applying the median filtering of the original image **a**).

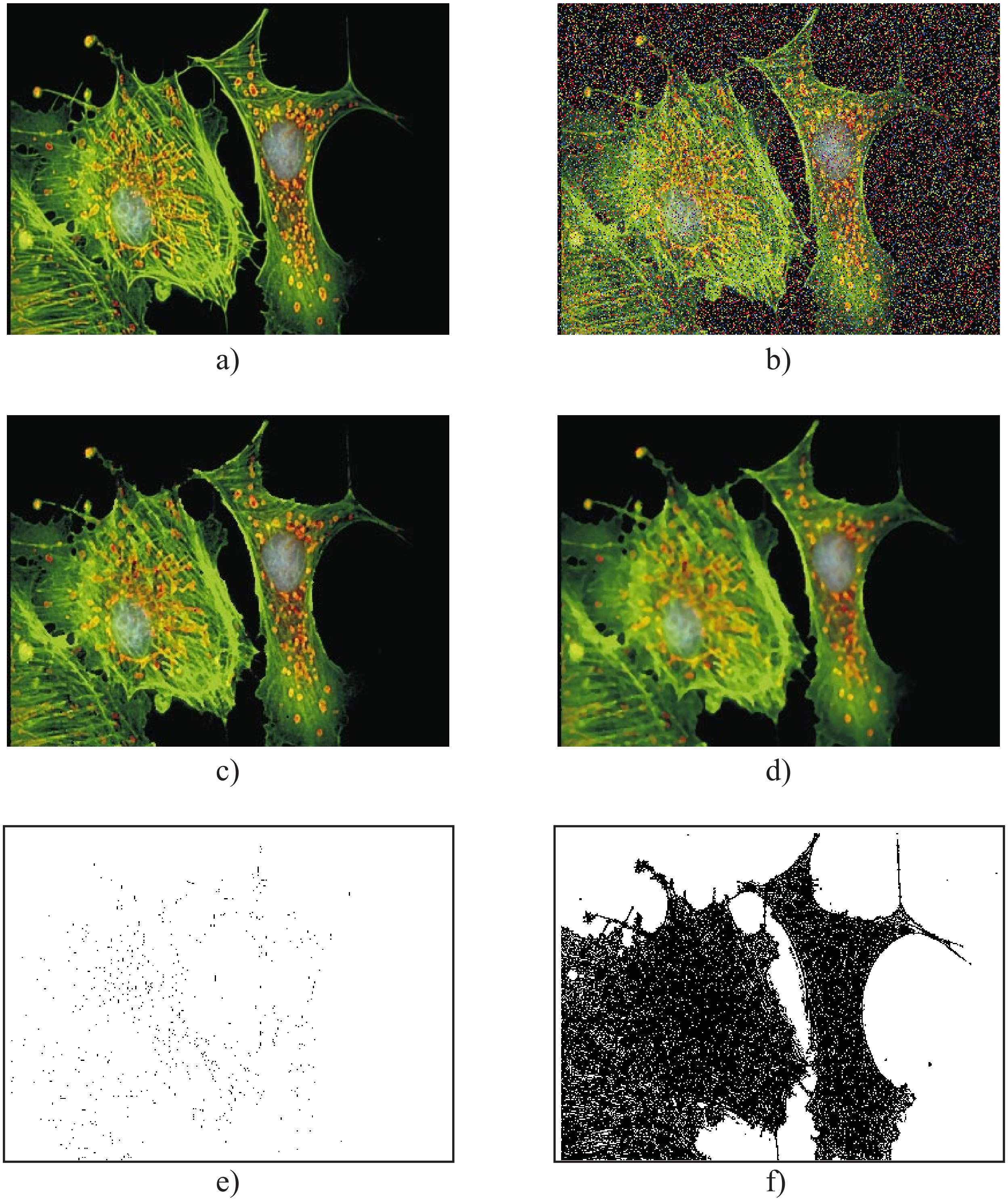

Figure 3: Comparison of the effect of the median and the proposed filter when applied on the noisy test image. **a**) test biological image, **b**) image distorted by 10 % impulse "salt & pepper" noise added to each RGB channels, **c**) noisy image filtered with the new method (vector algorithm, $\beta = 2$, 3 iterations), **d**) noisy image filtered with the 3×3 median - 3 iterations, **e**) all pixels whose colour values were changed by our filter when applied on the test image **a**) are marked black, the pixels with unchanged values after filtration are white, **f**) the same when applying the median filtering of the original image **a**).